Estimating Discrete Games*

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July 2011

Abstract

This paper provides a critical review of the methods for estimating static discrete games and their relevance for quantitative marketing. We discuss the various modeling approaches, alternative assumptions, and relevant trade-offs involved in taking these empirical methods to data. We consider both games of complete and incomplete information, examine the primary methods for dealing with the coherency problems introduced by multiplicity of equilibria, and provide concrete examples from the literature. We illustrate the mechanics of estimation using a real world example and provide the computer code and dataset with which to replicate our results.

Keywords: Discrete choice; Games estimation; Structural models

JEL Codes.: C1, C7, M3, L2

*Contact information: paul.ellickson@simon.rochester.edu, sanjog.misra@simon.rochester.edu. The authors would like to thank participants in the workshop on Quantitative Marketing and Structural Econometrics hosted by Duke University's Fuqua School of Business, Columbia Business School, and UCLA's Anderson School of Management. This paper benefitted immensely from the comments and suggestions of the editors and two anonymous referees. Any remaining errors and omissions are our own.

1 Introduction

Marketing is about understanding, predicting and influencing the behavior of consumers and firms. Each face many interrelated decisions. Individual consumers decide what brands to purchase, how much to buy, where to make their purchases and with whom to enjoy them. Firms choose which products to offer and what prices to charge, how to position and promote their brands, whom to hire and how to compensate them, and how much to invest in the continued growth of their enterprise. Most of these decisions involve strategic interactions: neither firms nor consumers act in a vacuum. Consumers may care about what their family and peers think of their choices, who else has purchased the product before them, whether it has received favorable reviews, the reputation of the seller, and whether complementary products are or will become available. Firms must consider the strategic reactions of the other players as well. Will a price increase be matched? Will consumers remain loyal to its products? How will its salespeople respond to incentives? Can a rival simply copy its business model? These are all strategic decisions.

The interrelated nature of these decisions suggests modeling them as strategic games. The precise structure of the game will clearly depend on the particular application. The game might be either static or dynamic, involve decisions (control variables) that are discrete or continuous (or mixed), and information settings that are either complete or incomplete. By estimating the structural parameters that govern these games, we can recover valuable information about the participants' payoffs (and costs) and make predictions concerning outcomes that are not observed in the data. Our focus here is on a particular subclass of strategic interactions: static discrete games. The purpose of this article is to summarize the current state of the art in analyzing these games, highlight the relevant trade-offs between alternative approaches, and identify areas that are ripe for further exploration. This is a decidedly applied piece, aimed at explaining *how* to estimate games as well as *why* they should be utilized. To ease the transition from learning to doing, we illustrate the nuts and bolts of estimation with a real world example: an entry game between Wal-Mart and Kmart. Using data on their actual choices, we construct estimators illustrating several canonical methods, and provide documented computer code with which to replicate our results.

We have chosen to focus on a narrow slice of the empirical games literature, namely static discrete games. Although the relevant decision variables are often continuous (e.g. prices, advertising), our focus on discrete actions is driven by three considerations. First, the empirical structure of discrete games is particularly complex, as it naturally involves decision rules that take the form of inequalities, as opposed to first order conditions. Thus, discrete games require a unique set of econometric tools. Second, many strategic decisions are naturally discrete (e.g. entry) and provide the only avenue by which to identify certain critical constructs (e.g. fixed costs). Third, the available data characterizing outcomes is often discrete, and it is useful to understand what we can and cannot learn from discrete choice data alone. Later, we will briefly consider what can be accomplished with richer data structures. Finally, our decision to focus on static games simply reflects the constraints of space and the desire to target an audience that is new to the study of games and looking for a reasonable jumping off point.

Our analysis follows the timeline of the literature, beginning with static games of complete information, and the pioneering work of Bresnahan and Reiss (1990, 1991) and Berry (1992). We first discuss the specific econometric problems that arise in games (of either information structure), namely that the interdependent nature of the underlying decision problems gives rise to multiple equilibria. This leads to a coherency problem in which the mapping from parameters to outcomes is non-unique, substantially complicating estimation. We discuss the four leading solutions to the coherency problem in detail, along with specific examples of each that are drawn from the extant literature. We conclude this first section with a working example implementing two canonical estimators on a real-world dataset.

Next, we turn to games of incomplete information, describing first how the information structure impacts the both the equilibrium concept and the method of solution. We then introduce the various methods of estimation, once again highlighting the role of multiplicity and its implications for correctly specifying the empirical model. We discuss several empirical examples from the literature and conclude by revisiting our working example under the alternative assumption of incomplete information. Finally, we conclude with a discussion of extensions to the basic empirical frameworks and directions for future research.

The paper is organized as follows. Section 2 provides a general introduction to empirical games, highlighting the various information and timing assumptions, and introduces our working example. Section 3 examines games of complete information. We discuss the primary methods of estimation, provide concrete examples from the literature, and illustrate implementation using our working dataset. Section 4 considers incomplete information games. We highlight the various estimation methods, review applications from the literature, and return to our working example to illustrate the mechanics. Section 5 considers extensions to the baseline models and directions for future research. Section 6 concludes.

2 A Taxonomy of Discrete Games

Discrete games concern choices made from a finite set of alternatives, where the payoffs from making each choice depend on the decisions of other players. That is, they are discrete choice models with strategic interactions. The canonical example is entry into a market, but other applications have included the timing of radio commercials, a supermarket's choice of pricing strategy, an ice cream manufacturer's choice of flavors, and an individual's decision to join a group. While researchers sometimes have access to richer data (beyond a discrete choice of action) such as price, quantity or cost information, most applications to date have focused on pure discrete choice data and employed a latent payoff structure, relying on revealed preference arguments to motivate the analysis. We will consider richer data structures later, but for now will assume that all choices are discrete and payoffs purely latent.

A critical consideration when formulating a discrete game involves specifying each player's information set and relevant time horizon (as well as what the researcher observes and does not observe). With regard to the players' information sets, there are two main approaches: complete information and incomplete information. Under the complete information setting, the researcher assumes that the players observe everything about each other's payoffs (including any covariates that are unobserved by the researcher) and therefore face no uncertainty regarding the payoffs of their rivals. The relevant equilibrium concept is Nash equilibrium and the standard approach is to focus on pure strategies.[1] Under the incomplete information setting, the players are instead uncertain about the payoffs and actions of their rivals. They form expectations over their rivals actions and maximize expected profits. The relevant equilibrium concept is then Bayesian Nash equilibrium and

[1] Mixed strategies are straightforward to handle in principle, but raise considerable complications in practice (e.g. they are difficult to solve for).

standard purification arguments imply that we need only focus on pure strategy equilibria.[2]

Turning to the player's relevant time horizon, there are again two main alternatives: assume they are playing a one-shot, static game or formulate an infinite horizon dynamic game.[3] For the purposes of this article, we will focus exclusively on the static, simultaneous move setting, referring the reader to excellent surveys by Ackerberg, Benkard, Berry, and Pakes (2005) and Aguirregabiria and Mira (2010) detailing the various approaches for estimating dynamic games. However, we note here that many of the issues that arise in the estimation of static games (e.g. multiplicity of equilibria, coherency problems, computational complexity) occur in the dynamic setting as well and several approaches to estimation (e.g. nested fixed point estimation, two-step methods) can be applied in either context. Indeed, the main methods for estimating static games of incomplete information were imported from the dynamic games literature.

2.1 Wal-Mart and Kmart Entry Game: A Working Example

To illustrate the various assumptions, modeling alternatives, and estimation methods available for static games, we will focus on a single, "real world" research example: an entry game between Wal-Mart and Kmart discount stores. To set the stage, suppose that Kmart and Wal-Mart compete in a collection of well-defined local markets (e.g. rural villages and small towns). Since we are focusing on small, rural markets, we will ignore the existence of Target, which mainly serves more urban locations. While their stores were actually sited over a 40 or 50 year period, we will assume that their strategic choice of locations can be well-approximated by a static discrete game.

We will draw on a dataset collected by Panle Jia for her empirical analysis of the discount retail industry (the dataset is publicly available on the *Econometrica* website and described in detail in Jia (2008)). We consider a (much) simplified version of her model, in which the two chains make independent entry decisions across a collection of local markets.[4] In

[2] Mixed strategies are typically introduced to alleviate concerns over existence of equilbrium. However, as noted by Harsanyi (1973), the crucial issue for existence is introducing uncertainty over rival choices. This uncertainty arises naturally with the presence of incomplete information.

[3] Note that these two "alternatives" are simply what *has been done* (at least in the bulk of the literature), as opposed to *what could be done* (e.g. alternating moves, repeated games, etc.).

[4] Assuming that firms make independent decisions across markets is clearly counterfactual for a chain of stores, but relaxing this assumption introduces a complex network structure to the choice problem. This network game is the focus of Jia's paper and an issue will we return to later.

addition, we (like her) consider only markets in which each firm operates at most one store. Taking a local market to be a county, this leaves 2,065 relatively small and isolated markets, assumed to be independent replications of this simple 2×2 discrete game (two firms choosing either "enter" or "don't enter"). We will consider each information setting (complete and incomplete) in turn, focusing on complete information first. We will then use this dataset to illustrate several specific estimation routines. The code and associated documentation are available online.

3 Complete Information Approach

The estimation of discrete games relies on the same revealed preference logic as discrete choice: the choice the firm actually made must have yielded higher profits (or expected profits) than the alternatives that it did not choose, conditional on the equilibrium choices of its rivals. The inclusion of rival choices as conditioning arguments in the players' payoff functions is what distinguishes discrete games from single agent discrete choice problems, introducing econometric complications that we will tackle shortly. Structural models of these strategic discrete choices provide insight into the drivers of profitably, both observed and unobserved. We follow the bulk of the literature in treating the payoffs upon which firms base their decisions as latent. The choice of functional form for these latent payoffs is clearly important, having direct implications for both tractability and the interpretation of results.

There are two main alternatives when choosing a functional form for payoffs: derive it from particular assumptions on the economic primitives (e.g. demand and cost, as well as the structure of the post-entry game) or choose a parameterization which is analytically convenient, yet flexible enough to capture the patterns observed in the data. While the first is clearly theoretically cleaner (all the parameters will have a clear structural interpretation), it can quickly become unwieldy, rendering estimation intractable. Moreover, absent data on prices and quantities, the identification of more primitive demand and cost structures will clearly be driven by functional form. As such, the second approach has become the de facto standard since Berry (1992). However, this more "reduced form" approach does place limitations on the causal interpretation of coefficients and the scope for performing counterfactuals.[5]

Following the structure of Berry (1992), but employing the notation of Ciliberto and Tamer (2009), let the profit function of firm $i = \{K, W\}$ in local market m be given by $\pi_{im}(\theta; y_{-im})$ where y_{im} is the action (enter or do not enter) of firm i, y_{-im} is the action of its rivals (just one rival firm, in our working example), and θ is a finite-dimensional parameter vector. The function π_{im} will typically contain covariates specific to both the market and the firms (e.g. population and the distance to the nearest distribution center). In particular, let X_m be a vector of market characteristics common to both firms and $Z_m = (Z_{Km}, Z_{Wm})$ represent firm characteristics which enter only into the focal firm's profit function (e.g. cost variables) and do not (directly) impact the profits of its rivals (other than through their impact on these rivals' entry decisions). In general, we might also consider firm characteristics that enter rival firm's profit functions as well (e.g. quality), but will focus on a more parsimonious setting here. Let the profit function of firm i in market m be given by

$$\pi_{im} = \alpha_i' X_m + \beta_i' Z_{im} + \delta_i y_{-im} + \varepsilon_{im} \tag{1}$$

where ε_{im} is a component of profits that is unobservable to the econometrician. Thus, expected profits (net of ε_{im}) are a function of only the common market characteristics, the firm's own characteristics, and its rival's chosen action. The assumption that the ε's are perfectly observed by both players makes this a game of complete information. Assuming that the firms make choices simultaneously, the complete information Nash equilibrium can be characterized by the following system of inequalities

$$\begin{aligned} y_{Km} &= 1\left[\alpha_K' X_m + \beta_K' Z_{Km} + \delta_K y_{Wm} + \varepsilon_{Km} \geq 0\right] \\ y_{Wm} &= 1\left[\alpha_W' X_m + \beta_W' Z_{Wm} + \delta_W y_{Km} + \varepsilon_{Wm} \geq 0\right] \end{aligned} \tag{2}$$

which, in this case, represents the non-negative profit conditions for Kmart and Wal-Mart respectively. An equilibrium is simply a configuration that satisfies both equations. Note that these outcome equations constitute a binary simultaneous equation system. The presence of a rival's choice variables on the right hand side of each firm's profit function are what distinguishes discrete games from discrete choice problems. This interdependent structure raises problems for estimation and identification that we discuss next.

[5] See Berry and Reiss (2007) for further discussion of the relevant trade-offs.

In particular, if the ε's have full support, it is straightforward to establish the existence of multiple equilibria. Put simply, this implies that for a given set of parameters there may be more than one possible vector of equilibrium outcomes (y). For example, if the δ's are assumed to be negative (facing competition reduces your profits), multiple equilibria arise in the region of ε space for which $-\left(\alpha_i'X + \beta_i'Z_i\right) \leq \varepsilon_i \leq -\left(\alpha_i'X + \beta_i'Z_i\right) - \delta_{3-i}$ for $i = 1, 2$. Intuitively, this represents the settings in which a local market can only "fit" one firm and neither firm's monopoly profits are large enough to preempt entry by the other (e.g. each firm's monopoly profits are only slightly greater than zero, so neither one wants to be there if the other one is).[6] As mentioned earlier, the same set of parameters (and covariates) are consistent with more than one outcome. This "incompleteness" raises a problem for inference known as coherency (Heckman (1978), Tamer (2003)). From a practical standpoint, in the simple 2×2 game considered above, the likelihood for the individual firm's choice probabilities will sum to more than 1, violating the law of total probability.

To date, there are four main approaches to "solving" the coherency problems raised by the multiplicity of equilibria: aggregate to a different set of predictions which are robust to multiplicity (e.g. the number of entrants), place restrictions on the model which guarantee a unique prediction (e.g. sequential moves), specify an equilibrium selection rule (e.g. the equilibrium maximizes joint profits), or embrace the ambiguity and adopt a bounds approach.[7] We will consider each strategy in detail, and then turn to the mechanics of estimation.

The strategy of aggregating up to a robust prediction was first proposed by Bresnahan and Reiss (1991), who developed a general framework for estimating discrete games and social interaction models. The core idea can be illustrated using the 2×2 Wal-Mart/Kmart entry game considered above. Note that in the region of ε space in which multiple equilibria arise, the multiplicity is in the *identity* rather than the *number* of entrants. In particular,

[6] For an elegant graphical illustration of this case see either Bresnahan and Reiss (1991) or Ciliberto and Tamer (2009). Note that a similar result obtains when the δ's are positive (i.e. entry is beneficial, as in a coordination game or peer-effects model), only now, in the region of non-uniqueness, either both players will enter or both will stay out.

[7] Note that even if one is able to "solve" the coherency problem and obtain consistent parameter estimates, multiplicity of equilibria may continue to raise difficulties at the counterfactual stage. For example, the selection rule that characterized the data may no longer be valid under the counterfactual.

either Wal-Mart or Kmart can profitably enter the market, but not both. There will be one entrant in equilibrium, but the model does not specify who it will be. Therefore, rather than specifying an econometric model that predicts *which firms* will enter, we instead construct a model of *how many firms* will enter. Given certain restrictions on the payoff functions (mainly restricting the degree of heterogeneity in payoffs) this strategy can be extended beyond the simple 2×2 setting, and likelihood functions written down that characterize the equilibrium number of entrants rather than the particular choices of individual players (Bresnahan and Reiss (1991), Berry (1992), Mazzeo (2002b)). However, with sufficient amounts of firm heterogeneity, it can be difficult to guarantee uniqueness in the number of entrants (or even the existence of pure strategy equilibria). Therefore, it is necessary to consider alternative approaches.

The second approach to completing the model involves changing the timing element of the model so that players move sequentially, rather than simultaneously. This sequential-move structure guarantees a unique equilibrium. In particular, when the parameters of the model fall in the region yielding the formerly ambiguous predictions, the "first mover" will preempt the follower, restoring coherency and allowing the likelihood to once again sum to one. This approach is employed by Berry (1992), where the operative assumption is that the most profitable firms enter first. This has the added benefit of mitigating the inefficient entry that might occur by simply assuming that Kmart always moves first, for example. Note that this approach can complicate estimation somewhat as the regions of integration - the partitions of ε space that yield each unique prediction - may have irregular shapes (i.e. be non-rectangular). Berry (1992) addressed this problem via simulation, which we will illustrate in detail below.

Clearly, sequential entry can be viewed as a form of equilibrium selection, albeit one that is imposed by the researcher. An alternative approach to completing the empirical model is to specify a more general selection rule that's a function of covariates (and perhaps unobservables). This approach was originally proposed by Bjorn and Vuong (1985), and further explored by Tamer (2003) and Bajari, Hong, and Ryan (2010), all in the context of complete information games. In the simplest version, this could involve assigning probabilities π and $1 - \pi$ to the two monopoly outcomes in the region of non-uniqueness and estimating this additional parameter (π) as part of an overall likelihood function. Note that the overall likelihood will now be a mixture. More generally, these probabilities might be allowed to depend on covariates (and perhaps the unobservables), leading to more complex mixture models. Alternatively, in Bajari, Hong, and Ryan (2010), the equilibrium selection probabilities depend on the property of the equilibrium itself (that it is joint profit maximizing, for example).

The fourth solution to the multiplicity problem is to embrace the incompleteness and switch to a bounds approach (Tamer (2003), Ciliberto and Tamer (2009), Pakes, Porter, Ho, and Ishii (2005)). Under this approach, the selection rule is viewed as an infinite dimensional nuisance parameter - an unknown function of unknown covariates. Rather than specifying a particular selection rule, the researcher seeks instead to identify parameters that are consistent with at least one such rule. While it may still be possible to achieve point identification using "identification at infinity" arguments (Ciliberto and Tamer (2009)), these models will generally be set identified. Establishing valid (and tractable) methods of inference for set identified models is an active area of current econometric research.

3.1 Unobserved Heterogeneity

In any empirical model, it is important to control for unobserved heterogeneity: features of the market or market participants that are unobserved to the researcher. In the context of entry games, an obvious example is the level of intrinsic demand, which is often poorly proxied by observables like population and income. Some markets are simply better locations, as they are closer to shopping districts, highway interchanges, or other local amenities. Many of these features will be difficult to capture with available covariates, forcing the researcher to deal with them econometrically.

The primary approaches to estimating static games of complete information involve a "full-solution" approach whereby, for a given guess of the relevant parameter vector, the game is first solved (for either the equilibrium number of entrants or individual choice probabilities, conditional on a particular selection mechanism) and then its predictions matched to what is observed in the data. This is essentially a full-information approach, allowing estimation to proceed either via maximum likelihood (MLE) or the generalized method of moments (GMM), perhaps employing simulation methods to reduce the computational burden of computing various high-dimensional integrals. Either way, it is relatively straightforward to include a rich structure of unobserved heterogeneity (e.g. market level random effects, random coefficients, etc.), provided that its inclusion does not violate the conditions necessary for completing the model (e.g. uniqueness of the equilibrium or the number of entrants). Furthermore, so long as the full data generating process can be specified parametrically, Bayesian inference is feasible as well, providing an attractive avenue for including heterogeneity at relatively low computational cost. The further exploration of a Bayesian approach to games estimation is a fertile area for future research.

3.2 Examples from the Literature/Extensions

Complete information models have been employed extensively in both the economics and marketing literatures, starting with the seminal work of Bresnahan and Reiss (1991) and Berry (1992). Mazzeo (2002b) extended the Bresnahan and Reiss approach to include a discrete choice of product quality, in addition to a binary entry decision. His application was to motels located along interstate highways. Consistent with standard predictions from oligopoly theory, he found that competition was strongest amongst the closest types. Cleeren, Verboven, Dekimpe, and Gielens (2010) use this approach to study intra- and inter-format competition among discounters and supermarkets. Zhu, Singh, and Manuszak (2009) adapted Mazzeo's framework to analyze entry and format choice in the discount retail store industry, using the selection correction techniques developed in Mazzeo (2002a) to include additional information on store level revenue. Singh and Zhu (2008) examined the impact of market structure on posted prices in airport rental car markets using a similar framework.

Hartmann (2010) developed a complete information framework for incorporating social interactions into marketing mix decisions. Clearly, within-group interaction should influence optimal targeting. His application is an individual golfer's discrete decision over whether to play a given round of golf alone or join a foursome. He also incorporates individual level heterogeneity through a hierarchical Bayesian Markov Chain Monte Carlo (MCMC) approach. Shriver (2010) extends the Bresnahan and Reiss framework to accommodate an endogenously determined market size in his model of indirect network effects in alternative fuel adoption. Ciliberto and Tamer (2009) use a bounds approach to examine airline entry decisions.

In the context of store locations, Jia (2008) relaxed the assumption of independence across markets by tackling the store network choice directly, exploiting a lattice structure that arises in the two player model. She is able to quantify the relative importance of network economies as well as the impact of Wal-Mart on small firms. However, her approach is only able to accommodate two firms (Wal-Mart and Kmart) that each operate only one store per market. Ellickson, Houghton, and Timmins (2010) use a profit inequalities approach similar to Pakes, Porter, Ho, and Ishii (2005) to accommodate multiple firms, an arbitrary number of stores per location, and a location-specific unobservable. They apply their framework to competition among Wal-Mart, Kmart, and Target, and highlight the importance of controlling for unobserved heterogeneity.

3.3 Implementation

To illustrate the complete information approach to static games, we will implement two of the strategies discussed above: aggregating up to a unique prediction and specifying a particular selection rule (based on the order of entry). The first approach is based on Bresnahan and Reiss (1991), while the second follows Berry (1992).

Model 1 (Bresnahan and Reiss): The first model we estimate is based on Bresnahan and Reiss (1991). We assume that firms are exchangeable and profits depend only on market level factors

$$\pi_{im} = \alpha' X_m - \delta y_{-im} + \varepsilon_{im} \tag{3}$$

In our application, the market level matrix X_m includes three covariates: "Population," "Retail sales per capita," and a dummy for "Urban" markets.[8] We will assume throughout that the ε_{im}'s are i.i.d. standard normal deviates.[9] Given this structure, the likelihood of observing n_m firms in a given market m can be computed in closed form. For example, the probability of seeing a duopoly is simply

$$\Pr\left(n_m = 2\right) = \prod_i \Pr\left(\alpha' X_m - \delta y_{-im} + \varepsilon_{im} \geq 0\right). \tag{4}$$

[8] For a detailed discussion of the industry, market definition, and the relevant covariates, see Jia (2008).

[9] As noted earlier, the complete information approach can easily accommodate both correlated errors and unobserved heterogeneity. However, for expositional simplicity and ease of comparison (to the incomplete information examples), we restrict our attention to the i.i.d. setting.

The sample log-likelihood is then

$$\ln \mathcal{L} = \sum_{m=1}^{M} \sum_{l=0}^{2} 1\left(n_m = l\right) \ln \Pr\left(n_m = l\right). \tag{5}$$

Estimation is carried out using full-information maximum likelihood (FIML). Results for all complete information games are presented in Table 1. We defer a discussion of the results until after we have introduced the remaining models.

Model 2-4 (Berry): We implement three versions of Berry's estimation framework, which also accounts for observed heterogeneity across players. The key distinction between these three cases is the way they resolve the multiplicity problem. The first version (Model 2) follows Berry (1992) in ordering entry by profitability. That is, the most profitable player moves first. The estimation algorithm, proposed by Berry (1992) involves simulating ε's to construct profits (from (1)) and then using these realizations to construct the profit realizations that order the moves. The second version (Model 3) gives Wal-Mart the option to enter first (independent of profits), while the third version (Model 4) awards Kmart this right.

Note that, given a particular order of entry, the number of firms in the market is uniquely determined. Since we know each firm's profitability (for a given ε), we also have a unique prediction of who will be in and out of the market. This information can then be used to construct an estimator. In our implementation, we follow the approach described in the appendix to Berry (1992). We construct the probabilities that Wal-Mart and Kmart will each enter the market by integrating over indicator functions describing entry as a function of computed profits and the equilibrium number of firms for each simulation. These probabilities can then used to construct moment conditions which define a GMM type objective function or to formulate a likelihood. In our implementation we adopt the latter strategy. We would like to caution the reader that formulating a likelihood for such games with more than two players or with added heterogeneity is not trivial. In such cases one would have to typically adopt a GMM approach and face a choice of what moments to include.

3.3.1 Discussion of Results

The results for Models 1 through 4 are presented in Table 1. All parameter estimates are significant at the 5% level, with signs that are consistent both with intuition and previous results based on the same data (Jia (2008)). Since the purpose of this empirical exercise is to demonstrate the different methodologies, we will not dwell upon the substantive aspects of the results here. Rather, we focus the reader's attention on the differences in the parameter estimates across the models. First, a warning - the results cannot be directly compared across the two sets of models (B&R vs. Berry) because of the usual concerns about scaling, specification, and assumptions. However, comparison across the three Berry models is fair and we restrict our attention to these.

At first blush, the results may seem "surprising" in that the parameter estimates are essentially identical across the three equilibrium selection rules: the particular rule does not seem to matter here. We note that Jia (2008) finds a similar robustness to order of moves in her analysis. To help explain why these results obtain, we remind the reader that this industry (at least for these data and this time period) has very asymmetric players. In particular, WalMart is quite dominant player while Kmart is relatively weak. The table below outlines this asymmetry in stark fashion:

	Kmart not in market	Kmart in market
Wal-Mart not in market	1004	90
Wal-Mart in market	711	260

Note that in only 4.35% of all markets does Kmart enjoy a local monopoly. Now consider how the assumed sequence of moves might impact the estimates. If we assume that Wal-Mart has the right to move first, the model must rationalize the fact that Wal-Mart enters about half the markets while Kmart enters much fewer. It does so by making Wal-Mart relatively more profitable, choosing the intercepts and covariates (especially the exclusions) such that Wal-Mart has a relative advantage. It should then come as no surprise that changing the rule to allow the most profitable firm to move first changes very little: the model continues to infer that Wal-Mart is the more profitable player. Now consider the case where Kmart moves first. The model can use similar parameters to rationalize the data since Kmart ends up acquiescing the vast majority of the markets to Wal-Mart. In other words, Kmart does not enter 'monopoly' markets because it expects the more profitable

Wal-Mart to do so. Since these markets can only sustain one player, Kmart does not enter. Again, to generate the patterns in the data it suffices to make Wal-Mart dominant as in the other cases.

We would like to be clear that this is not a general result but rather contingent on the particular data at hand. Other applications and industries will have varying number of players with different power structures and the equilibrium selection rule may indeed come to have some 'bite.' It is also important to note that the choice of the payoff specification is key. If, for example, we allowed the players to have different coefficients across the board (particularly δ) the sequence of moves would have a significant impact on estimates. In general, modelling assumptions in discrete games (especially those pertaining to equilibria) are not trivial and can have important influence on parameter estimates. This is part of the motivation for the bounds approach.

4 Incomplete Information Approach

Beginning with Rust (1994), a parallel literature has emphasized static games of incomplete information. Under the incomplete information approach, payoffs are no longer assumed to be common knowledge: players form beliefs over rivals' actions. While uncertainty can be introduced in a number of ways, the easiest way to model incomplete information is through the ε's, the additively separable components of payoffs that are unobserved to the researcher. In particular, suppose we now assume that each player observes its own ε_i, but only knows the distribution of ε_j for its rivals, which we denote $F(\varepsilon_j)$. Suppose that the researcher also knows this distribution, but does not observe individual draws for any player. Note that this puts the firms on equal footing with the researcher regarding the beliefs over their rivals' actions, a symmetry which will prove very useful when constructing an estimator. Each firm now forms expectations about its rivals' behavior, choosing the action that maximizes expected profits given those beliefs. This yields the following system of inequalities

$$\begin{aligned} y_{Km} &= 1\left[\alpha_K' X_m + \beta_K' Z_{Km} + \delta_K p_W + \varepsilon_{Km} \geq 0\right] \\ y_{Wm} &= 1\left[\alpha_W' X_m + \beta_W' Z_{Wm} + \delta_W p_K + \varepsilon_{Wm} \geq 0\right] \end{aligned} \tag{6}$$

in which the probability $p_i \equiv E_i(y_{-i})$ represents firm i's beliefs regarding its rival's actions. The conditional choice probabilities implied by these decision rules can then be used to represent each firm's strategy. The Bayesian Nash equilibrium (BNE) of the game can be characterized by the following set of equalities

$$\begin{aligned} p_K &= \Psi_K(\alpha_K' X_m + \beta_K' Z_{Km} + \delta_K p_W) \\ p_W &= \Psi_W(\alpha_W' X_m + \beta_W' Z_{Km} + \delta_W p_K). \end{aligned} \tag{7}$$

where the exact form of Ψ will depend on the distribution F. The functions Ψ are best response probability functions, mapping expected profits (conditional on beliefs p) into (ex ante) choice probabilities.[10] If F is an absolutely continuous distribution, this system (pair, in this case) of nonlinear equations is guaranteed to have a solution by Brouwer's fixed point theorem. Moreover, this fixed point representation provides a direct method of solving for equilibria: the method of successive approximations (i.e. fixed point iteration). Thus, one possible estimation strategy (originally proposed by Rust (1994)) is another "full solution" approach that first solves this system of equations (for a given set of parameters) and then matches the predicted conditional choice probabilities to the choices observed in the data. This is essentially a static games version of Rust's nested fixed point (NFXP) algorithm (Rust (1987)). If the ε's are assumed to be drawn from the Type 1 Extreme Value distribution, the likelihood function will take the familiar conditional logit form. This is the method employed by Seim (2006) in her empirical study of entry and store location decisions in the video rental industry (though she uses a nested logit structure to distinguish entry from location choice).

A now familiar complication is that the system of equations (7) may admit more than one solution: the underlying game may have multiple equilibria.[11] Thus, games of incomplete information suffer from the same coherency problems as complete information games. Once again, there are several possible methods of completing the empirical model. We will first

[10] In the incomplete information setting, strategies can be represented as either discrete actions or ex ante choice probabilities. The modifier ex ante refers to the fact that these probabilities constitute the firm's expected actions prior to the realization of ε.

[11] In some examples, incomplete information has been shown to reduce the incidence of multiple equilibria relative to a complete information counterpart. However, it does not eliminate the problem in general (see Berry and Reiss (2007) for a numerical example), implying that additional structure will still be needed to close the model.

review the four approaches discussed earlier, and then introduce two additional options that specifically exploit the structure of incomplete information games.

As with games of complete information, one strategy is to identify a prediction of the model that is robust to multiplicity. Since this becomes quite difficult in the presence of heterogeneity, this approach has not been pursued in the existing literature. The second option is to change the timing of the game from simultaneous to sequential moves. In his analysis of movie release dates, Einav (2010) employed a sequential structure, and provided a clever method for "integrating out" over alternative move sequences. The third option is to specify an explicit equilibrium selection mechanism in the spirit of Bjorn and Vuong (1985). This approach is developed further by Sweeting (2009) in his study of the timing of radio commercials. In his empirical model, Sweeting first considers cases in which the selection probabilities are fixed parameters and then a richer specification in which they depend on covariates. He also demonstrates that multiplicity of equilibria in the data can actually aid identification by changing the implied dispersion of choice probabilities. Misra (2008) proposes a Bayesian approach to estimation that uses MCMC to sample from the posterior distribution of the structural parameters, eliminating the need to search for all the fixed points.

Finally, if the researcher is unwilling to impose an explicit selection rule, a bounds approach may be feasible here as well. Since the moment inequalities approach developed by Pakes, Porter, Ho, and Ishii (2005) is robust to alternative assumptions on the players' information sets, it can be applied to either games of complete or incomplete information. Grieco (2010) develops an alternative framework which is also able to nest both information assumptions, along with an econometric test that can distinguish between the two.

The first four approaches to completing the empirical model are familiar from our discussion of complete information. However, the specific structure of incomplete information offers some additional options for completing the empirical model. The first, which draws on methods originally introduced by Hotz and Miller (1993) in the dynamic discrete choice literature, involves substituting first-stage estimates ($\hat{p}_i$) of the (reduced form) choice probabilities into the right hand side of equation (7). Note that this eliminates the need to solve the fixed point problem when evaluating the corresponding (pseudo) likelihood function that is implied by these structural choice probabilities.[12] Closely related "two-step" methods have proven very effective in estimating dynamic discrete games where, in addition to the problems raised by multiplicity, there is also a massive computational burden induced by the curse of dimensionality inherent to many dynamic decision problems.[13] In the case of static games, the primary benefit of the two-step approach lies in its relative robustness to multiplicity. Provided that only one equilibrium is played in the data, this solves the coherency problem since the estimator is effectively able to "condition on the equilibrium that was played in the data". This clearly relies on the condition that only one equilibrium *was* in fact played in the data. This is more likely to hold in settings in which the same set of firms compete over time in the same market versus settings where they compete in different markets (i.e. panel versus cross section).[14] With panel data, it may be possible to estimate the model market by market, allowing for the possibility that different equilibria are played in different markets (Ellickson and Misra (2008b)), thereby weakening the "one equilibrium" assumption.

Two-step methods do have some drawbacks relative to the full solution approaches discussed above. First, since they are inherently limited information techniques, they are less efficient than full-information maximum likelihood (FIML) approaches like Rust's NFXP estimator. Second, the consistency of the second stage estimates relies on obtaining consistent first-stage estimates of the conditional choice probabilities (CCPs). Since these are reduced form objects, they should ideally be estimated non-parametrically. This is due to the fact that, even if the functional form of both the profit functions (1) and best response probability functions (7) are known, the *reduced form* CCPs (i.e. the *solution* to equation (7)) will typically not be (hence the need for fixed point iteration). Since non-parametric estimation suffers from a well-known curse of dimensionality, it is likely that any first stage estimates of the CCPs will be quite noisy, yielding small sample biases in the second stage

[12] Note that GMM or least squares based estimation can be used here as well. See Bajari, Hong, Krainer, and Nekipelov (2010) for further details, as well as formal results on identification.

[13] As noted earlier, we have chosen to focus only on static discrete games. See Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008) for the seminal papers applying two-step estimation techniques to games. Arcidiacono and Ellickson (2011) provide a broad overview of two-step approaches to dynamic decision problems.

[14] Of course, in settings in which the same firms are observed over many periods, one might start to worry about linkages across time and the need to control for dynamics.

(structural) parameter estimates.[15] The bias will persist asymptotically if the researcher resorts to a parametric first stage, due to misspecification. Aguirregabiria and Mira (2002, 2007) have proposed a recursive extension of the two-step pseudo likelihood estimator that mitigates the small sample bias (and eliminates the requirement of a consistent first stage) by iterating on the best response probability mapping (7). This effectively swaps the order of the nests in Rust's NFXP approach. By forcing the conditions for a BNE to be satisfied, this nested pseudo likelihood (NPL) estimator is fully efficient (i.e. equivalent to FIML), so long as it converges. However, NPL relies on best response iteration, and therefore cannot find equilibria that are not best reply stable (Pesendorfer and Schmidt-Dengler (2010), Su and Judd (2007)). As such, it is not guaranteed to converge. Nonetheless, it has been found to work well in several applications (Aguirregabiria and Mira (2007), Ellickson and Misra (2008b)).

The final estimation method is the constrained optimization approach proposed by Su and Judd (2007). Su and Judd (2007) recast the unconstrained optimization problem described above as a constrained optimization problem subject to the equilibrium constraint (7), referring to their approach as a Mathematical Program with Equilibrium Constraints (MPEC). Since it does not rely on repeatedly solving for equilibria (or that the equilibrium constraints be satisfied at each point in the search process), MPEC is both computationally light and robust to best reply unstable equilibria. However, like NPL, it does impose a particular selection rule: the equilibrium that is played is the one that maximizes the likelihood.

4.1 Unobserved Heterogeneity

Another relevant trade-off in the choice between the various full-solution approaches (NFXP, NPL, MPEC) and the computationally lighter two-step procedure is the ability to accommodate unobserved heterogeneity. Since they are all essentially full-information approaches, it is relatively straightforward to account for heterogeneity using any of the full-solution methods. Of course, including random effects and/or random coefficients will clearly increase the computational burden and make the search for equilibria significantly more complex, though various simulation methods and Bayesian MCMC techniques could certainly be em-

[15] See Aguirregabiria and Mira (2007) and Su and Judd (2007) for relevant monte carlo evidence.

ployed here. Two-step approaches are much less accommodating, due to their more "limited information" structure and reliance on a non-parametric first stage. In particular, any unobservables that are conditioned on by the players must also be accounted for in the first stage estimation procedure. Since this is typically treated as a reduced form, this is not at all straightforward. For example, even if rival firms' unobserved state variables do not enter a given firm's payoff function (as is usually assumed to be the case), they generally will enter the reduced form CCPs and typically in a highly non-linear manner. This can make it very difficult to obtain consistent first stage estimates. Fortunately, there are several possible remedies.

First, as argued by Ellickson and Misra (2008b), if the researcher is willing to assume that the unobserved heterogeneity is private information, the non-parametric first stage CCPs will still be consistent, allowing the heterogeneous parameters to be integrated out of the second stage using standard simulation methods. Furthermore, if the unobserved heterogeneity occurs at the market level, the first stage could be estimated market by market (or even player by player, if there is enough data). This is the approach used in Ellickson and Misra (2008a) for their analysis of supermarket pricing strategies. Second, if one is willing to assume that the unobservables are a smooth function of observable covariates, a control function approach is also feasible (Bajari, Hong, Krainer, and Nekipelov (2010)). Finally, by enforcing the full structure of the model, the NPL approach can also be employed (eliminating the need for a consistent first stage and allowing for a rich structure on the unobservables). Of course, this is then equivalent to returning to a full-solution approach. Ellickson and Misra (2008b) use NPL to control for correlated, market level unobservables.

4.2 Examples from the Literature/Extensions

Several authors have used incomplete information games to shed light on issues of direct concern to marketers. Zhu and Singh (2009) employ Seim's nested fixed point approach to model entry and location decisions by Wal-Mart, Kmart and Target, documenting the importance of both heterogeneous competition effects and firm-specific preferences. Orhun (2006) extends Seim's approach to include location specific unobservables, applying her framework to the entry and location decisions of supermarkets. Ellickson and Misra (2008b) use both the two-step and NPL approaches to examine the strategic choice of pricing strategies in the supermarket industry. They find strong evidence of assortative matching - firms tend to coordinate on the same pricing strategy (e.g. EDLP or Hi-Lo) as their local rivals - and empirical results that support several specific predictions from marketing theory. Sudhir, Datta, and Talukdar (2007) employ a nested fixed point approach to examine the trade-offs between differentiation and agglomeration in the grocery industry.

Vitorino (2007) examines the joint entry decisions of stores in regional shopping centers, explicitly controlling for multiple equilibria using an MPEC approach. Draganska, Mazzeo, and Seim (2009) model the assortment decisions of ice cream manufacturers, incorporating information from a discrete choice demand system, while Musalem and Shin (2010) provide an alternative model of pricing and product line decisions.

4.3 Implementation

To illustrate the incomplete information approach to static games, we now implement several of the approaches described above using the same dataset as before. We note that, unlike the complete information approaches discussed earlier, each of which altered the underlying structure of the game, the methods considered here are all being applied to the exact same game (i.e. they are simply alternative estimators, not different structures). We begin with the full solution (nested fixed point) approach, and then illustrate the two-step and Nested Pseudo Likelihood (NPL) techniques.

Method 1 (NFXP): The first incomplete information framework we implement is the nested fixed point approach. We use the same profit specification as the complete information case

$$\pi_{im} = \alpha' X_m + \beta_i' Z_{im} - \delta y_{-im} + \varepsilon_{im}, \tag{8}$$

and include the same covariates as before. The ε's are again assumed to be i.i.d. standard normal (but treated as private information now). The estimation routine requires solving the following fixed point problem

$$p^*_{im} = \Phi(\alpha' X_m + \beta_i' Z_{im} - \delta p^*_{-im}) \tag{9}$$

which we accomplish via simple Picard iteration (successive approximation). We note here that, in keeping with the extant literature, we are not employing an exhaustive search for all possible fixed points.

Once the fixed point probabilities are obtained, they feed into a simple log likelihood

$$\ln \mathcal{L} = \sum_{m=1}^{M} \sum_{i \in \{W,K\}} y_{im} \ln(p^*_{im}) + (1 - y_{im}) \ln(1 - p^*_{im}) \tag{10}$$

which is then maximized to obtain parameter estimates.

Method 2 (2STEP): As noted earlier, the two-step estimator eliminates the need to solve for a fixed point by recognizing that, at the "true" solution, the probabilities are simply (unknown) functions of the covariates. In the first stage, we construct consistent estimators of these equilibrium conditional choice probabilities (CCPs). In principle, this first stage should be nonparametric. If, for some reason (such as inadequate data), nonparametric methods are infeasible, we suggest using a semi-parametric approach like the method of sieves or Generalized Additive Models (GAMs).[16] In our implementation, we use a GAM with tensor product interactions between the variables. This first stage yields fitted probabilities ($\hat{p}^{\{1\}}_{im}$) which we then "plug-in" to construct a likelihood

$$\ln \mathcal{L} = \sum_{m=1}^{M} \sum_{i \in \{W,K\}} y_{im} \ln\left(p^{\{2\}}_{im}\right) + (1 - y_{im}) \ln\left(1 - p^{\{2\}}_{im}\right) \tag{11}$$

in which

$$p^{\{2\}}_{im} = \Phi(\alpha' X_m + \beta'_i Z_{im} - \delta \hat{p}^{\{1\}}_{-im}) \tag{12}$$

where $\Phi(\cdot)$ is the standard normal CDF.

Method 3 (NPL): The Nested Pseudo Likelihood approach of Aguirregabiria and Mira (2007) iterates on the best response probability mapping (12) to reduce small sample bias (and eliminate the need for a consistent first stage). Note that we can always construct a new estimate of the CCPs from the best response mapping

$$\hat{p}^{\{k\}}_{im} = \Phi(\hat{\alpha}^{\{k\}\prime} X_m + \hat{\beta}^{\{k\}\prime}_i Z_{im} - \hat{\delta}^{\{k\}} \hat{p}^{\{k-1\})}_{-im}) \tag{13}$$

where $\left\{\hat{\alpha}^{\{k\}}, \hat{\beta}^{\{k\}}_i, \hat{\delta}^{\{k\}}\right\}$ are obtained by maximizing

$$\ln \mathcal{L} = \sum_{m=1}^{M} \sum_{i \in \{W,K\}} y_{im} \ln\left(p^{\{k-1\}}_{im}\right) + (1 - y_{im}) \ln\left(1 - p^{\{k-1\}}_{im}\right) \tag{14}$$

[16]For a comprehensive discussion of semi- and non-parametric methods (including sieves and GAMs), see Pagan and Ullah (1999).

The algorithm continues until $\left\| \hat{p}_{im}^{\{k\}} - \hat{p}_{im}^{\{k-1\}} \right\| \leq \epsilon$. In our implementation, we initialize the NPL estimator with the two-step probabilities (i.e. $\hat{p}_{im}^{\{1\}}$) and iterate until convergence $(\epsilon = 1\text{E-}8)$.

4.3.1 Discussion of Results

Results from each method are presented in Table 2. As we noted above, the three models represent different estimation approaches for the same underlying game. It is therefore comforting that the coefficients do not vary much across the columns. It is perhaps noteworthy that the 2STEP results are so close to the full information estimates in this case, suggesting that small sample bias is not an issue here. While it is tempting, we will refrain from speaking to the differences between the complete information and incomplete information results since they are obtained using very different assumptions and estimation algorithms. However, there has been some recent work on integrating and testing information structures in discrete games (see e.g. Aradillas-Lopez (2010), Grieco (2010), Navarro and Takahashi (2010)). In general, all estimates (across both tables) have the same sign and similar relative magnitudes (e.g. Wal-Mart has a higher intercept). Ultimately, the choice of modelling framework and estimation algorithm is left to the researcher.

5 Discussion and Future Directions

5.1 Complete versus Incomplete Information: Which Framework Makes More Sense?

The empirical relevance of complete versus incomplete information will clearly depend on the specifics of the particular application being considered. Is it more reasonable to assume that payoff functions are common knowledge or are there obvious sources of uncertainty? Advocates of the complete information approach note that static games are typically motivated as an approximation to long run equilibrium, at which point any uncertainty or randomness has long since been resolved. Thus, the assumption that players face no uncertainty and can perfectly predict what their opponents will do (ignoring the possibility of mixed strategies) may seem quite reasonable. Complete information games have also received more attention in the theory literature and their properties are better understood.

By contrast, under incomplete information, players cannot perfectly predict what their rivals will do - they behave as if they are playing against a distribution of player "types". Consequently, they may prefer to change their minds once they observe the actual decisions of their rivals. This is ruled out by the one-shot, simultaneous-move structure of the game. This vulnerability to ex post regret was first noted by Einav (2010), and was part of his motivation for changing the timing of the model to sequential moves (where such regret is mitigated). Of course, randomness and uncertainty seem a natural component of most strategic interactions. It is not hard to think of real world examples of firms who guessed wrong about the appeal of a new product (new Coke!) or the reaction of their rivals (HD DVD). Unfortunately, the one-shot structure of static games does not give players the ability to adjust to these realizations. This is a primary motivation for introducing dynamics, whereby firms are able to adjust to an ever-evolving flow of new information. While two-step methods have dramatically reduced the computational burden of estimating such models, the empirical analysis of dynamic discrete games is still at an early stage of development.

5.2 Beyond Latent Payoffs

We have thus far followed the bulk of the existing literature in considering purely latent payoff structures. This is frequently the most empirically relevant case, as discrete choices are often all that's observed in the data (furthermore, some choices, like entry, are naturally discrete). While a complete discussion of mixed continuous and discrete games is beyond the scope of this article, we will briefly discuss some recent methods for incorporating additional, post-entry information on quantities, prices, revenue or costs. Such data is increasingly available via direct partnerships between researchers and firms, as well as the proliferation of high quality academic datasets like the IRI Marketing Data Set. These data can be used to estimate more sophisticated specifications for the game. For example, if one had access to prices and market shares it might be possible to construct structural profit measures (ala BLP) and use those instead of their reduced form analogs.

Ignoring the information contained in post-entry outcomes is inefficient. It may also reduce the set of parameters that can be identified and limit the scope of any subsequent counterfactuals. Unfortunately, incorporating payoff data into discrete empirical games is not straightforward, as the researcher must now characterize the full joint distribution of *both* the choice data and whatever additional data he has chosen to incorporate. At the very least, this will dramatically increase the burden of solving for equilibria. Moreover, it also introduces a problem of non-random selection. For example, the same unobservables that lead a firm to charge a higher price (e.g. unobserved quality) will almost surely impact their entry decisions as well. A small but growing literature seeks to address these concerns.

The selection problem associated with incorporating outcome data was first noted by Reiss and Spiller (1989) in their model of airline competition, under the assumption of complete information. They propose a full solution approach to modeling the joint distribution of entry decisions and revenue outcomes, but place strong restrictions on the scope for strategic interaction. Drawing on the empirical labor economics literature, Mazzeo (2002a) used a first stage complete information game to construct a Heckman (1978) style control function in his study of the effect of market structure on equilibrium prices in the motel industry.[17] One drawback of this approach is its reliance on a purely statistical selection correction: the errors in the outcome equation are simply correlated with the errors in the choice equation. Ellickson and Misra (2008a) have recently proposed a propensity score based approach that allows the auxiliary data (revenue in their application) to depend directly on the same unobservables as the choice data.

5.3 Beyond Independent Markets

Up to now, we have assumed (alongside the bulk of the extant literature) that firms (players) make independent decisions across markets (choice situations). While this may be quite realistic in some settings (e.g. barber shops in rural villages), most actual applications have involved industries in which most of the players are national chains (e.g. discount stores, supermarkets, airlines, video and car rental outlets, gas stations). The associated "network choice problem" introduces several complexities, substantially increasing the computational burden and data requirements and exacerbating multiplicity problems. Nonetheless, there is a small and growing literature aimed at relaxing the independence assumption and directly tackling the formation of retail networks.

[17] Singh and Zhu (2008) and Zhu, Singh, and Manuszak (2009) apply Mazzeo's approach in alternative settings.

Jia (2008) developed a complete information framework for modeling spatial competition between two retail chains. By exploiting the supermodular structure of the two firm problem, she is able to substantially reduce the burden of solving for Nash equilibria, closing the model with an ex ante equilibrium selection rule. However, her elegant, lattice based solution method requires that the spillovers between own stores be positive and can only accommodate up to two players and a single outlet per location. Nishida (2008) extends Jia's approach to accommodate multiple outlets (but only two players). Ellickson, Houghton, and Timmins (2010) propose an alternative framework, based the profit inequalities approach of Pakes, Porter, Ho, and Ishii (2005), which can handle any number of players and spillovers of either sign. They do not require an equilibrium selection mechanism, but can only set identify many of the structural parameters. The structural analysis of network choice problems and complex spatial equilibria remains a fertile area for future research.

6 Conclusions

Discrete games offers an exciting avenue for marketing researchers. While we have focused our attention on static games, there are also new developments and challenges in dynamic games that should be of interest to marketers as well. This paper provides a critical overview of the estimation of static discrete games, aimed at providing a concise introduction for those who are interested in the field. We have also included computer code for implementing a few of the most basic examples, intended as a jumping off point for more complex and realistic implementations. We hope that our efforts will spur interest in the area and encourage researchers to add these concepts and methods to their toolkit.

Table 1: Estimation Results from Complete Information Games

Variable	B&R	Berry	Berry	Berry
	(Homogeneous)	(Profit)	(Wal-Mart)	(Kmart)
Common Effects				
Population	1.32*	1.69	1.67	1.69
Retail Sales per capita	1.13	1.54	1.52	1.54
Urban	1.03	1.20	1.19	1.20
δ	0.65	0.39	0.40	0.38
Wal-Mart Specific Effects				
Intercept (Wal-Mart)	-14.03**	-11.87	-11.76	-11.90
Distance to Bentonville, AK		-1.07	-1.06	-1.07
South		0.72	0.72	0.71
Kmart specific Effects				
Intercept (Kmart)	-14.03**	-19.76	-19.56	-19.57
MidWest		0.37	0.37	0.37

*All coefficients are significant at the 5% level.

**Intercepts are common across both firms in this specification.

Table 2: Estimation Results from Incomplete Information Games

Variable	NFXP	2STEP	NPL
Common Effects			
Population	1.90*	1.81	1.89
Retail Sales per capita	1.61	1.65	1.69
Urban	1.34	1.30	1.37
δ	1.10	0.92	1.63
Wal-Mart Specific Effects			
Intercept (Wal-Mart)	-13.17	-13.00	-13.80
Distance to Bentonville, AK	-1.03	-1.09	-1.04
South	0.58	0.67	0.63
Kmart specific Effects			
Intercept (Kmart)	-20.56	-20.75	-21.18
MidWest	0.34	0.31	0.30

*All coefficients are significant at the 5% level.

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Appendix: Glossary

- **Complete Information game:** A game in which each player's payoff function (the mapping from the full set of players actions to the focal players payoff) is common knowledge amongst all players.
- **Incomplete Information (aka Bayesian) game:** A game in which at least one player is uncertain about another player's payoff function.
- **Nash Equilibrium:** A strategy profile in which each player's strategy is a best response to their (correct) beliefs regarding rival play.
- **Bayesian Nash Equilibrium:** The Nash equilibrium of a Bayesian game.
- **Revealed preference:** The process by which a decision maker's preferences can be revealed through their choice behavior.
- **Coherency**: A coherent econometric model is one that yields a unique prediction for the endogenous (dependent) variables as a function of the observed and unobserved exogenous variables.
- **Incomplete model:** An econometric model in which the mapping from exogenous variables to endogenous outcomes is a correspondence, rather than a function.
- **Set (aka partial) identification:** An econometric model in which, even given access to infinite data, the parameters of interest cannot be point identified, but only found to lie within a non-singleton set. This often occurs when the researcher is unable (or unwilling) to impose assumptions strong enough to achieve point identification.
- **Equilibrium Selection Rule:** In a game with multiple equilibrium, a equilibrium selection rule is mechanism that specifies which equilibrium is actually played.
- **Control Function approach:** An econometric technique in which auxiliary variables are used to break the correlation between endogenous covariates and the outcome variables of interest.